Old Tests and New Curricula –

Can old assessment structures measure achievement in the digital mathematics curriculum?

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Defining the 'digital mathematics curriculum'

Can old assessment structures measure achievement in the digital mathematics curriculum? Or do we need to re-think the assessment of school mathematics in the digital age? This is an important question for us as test developers – but it is not a simple one.

Before we can consider the possible impact of the digital mathematics curriculum on assessment we need to identify some key features of that curriculum. What makes it different? One key aspect of the new curriculum is the use that can be made of computer graphics. In the United Kingdom, the impact of computer graphics on the teaching and learning of mathematics in primary and secondary school classrooms is increasing rapidly as more and more schools obtain access to an expanding range of mathematics teaching software. Computer graphics can be dynamic, interactive, and infinitely variable. They enable us to share mental images of structures, patterns and working systems that have hitherto been hidden away inside each individual's mind. They put into motion the flat, static 'snapshot' views of drawn or printed diagrams. As Kate Mackrell and Peter Johnston-Wilder remark,

> 'One of the ironies of trying to describe motion and its effects in text is that one necessarily has to miss out on *all* of the essential ingredients. Not least among these is the sense of surprise and wonder that animating mathematical diagrams and images can bring, externalising and setting back in motion images that have been held static in the pages of textbooks for over 2000 years.'
> Mackrell, K and Johnston-Wilder, P, 2005, p 82

The dominance of print as the defining factor of 'proper' mathematics is, at last, being challenged in the classroom. This change may need to be reflected in assessment. But just what difference does it make to what teachers teach or to what students learn?

Many teachers in the United Kingdom have found that a familiarity and facility with an ever-growing range of mathematics teaching software can help them to make key concepts accessible to a much wider range of students. Examples abound. Some of the most commonly reported are those that come from the use of dynamic geometry packages such as *Cabri Geometre* or *Geometer's Sketchpad*. Dynamic geometry software allows the user to construct diagrams that keep their defining features, while their dependent features can be varied at will. So, for example, a parallelogram may be constructed to have two pairs of parallel opposite edges of equal length, and two pairs of equal opposite angles. These properties of the construction will be maintained as the student drags a vertex of the parallelogram to vary its non-defining properties. The angles will become greater or smaller, and the sides will become longer or shorter, but each pair of opposite angles will remain equal, and each pair of opposite sides will remain both equal and parallel. In this way the parallelogram will change its shape, but it will still be a parallelogram. (For a simple, free example of this, go to http://www.mathsnet.net/dynamic/cindy/parallelogram.html. See Fig 1).

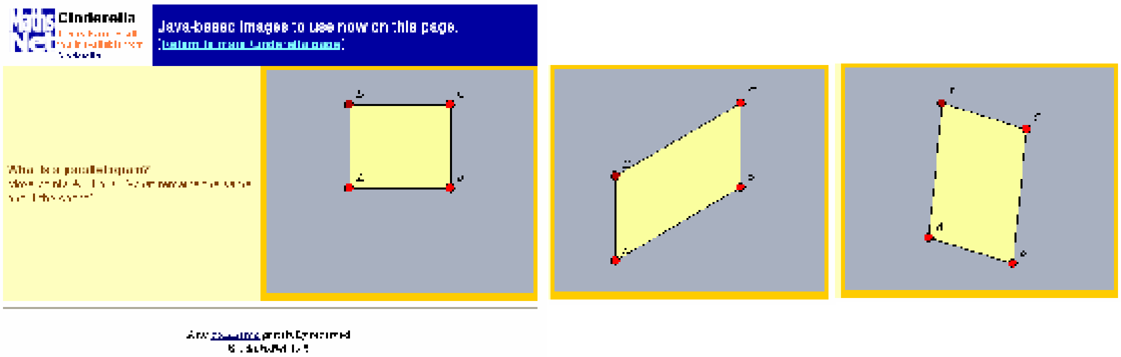


Fig 1 Example of a dynamic geometry image

The vertices of the rectangle can be dragged, and the angles and lengths of the sides changed, but the defining properties of the parallelogram are maintained.

In this way the student becomes familiar with parallelograms and other geometric shapes, and may develop a feel for their nature and their defining properties long before they learn the more formal definitions.

So, as Rubin argues, in a traditional print-based classroom,

> 'Most students' notions of geometry are, at worst, of two-column proofs that follow a series of arcane rules, illustrated by one or two static line drawings…. [Only those students who] enjoy and succeed in geometry are able to supplement these pictures with some sense of motion, e.g.: if this corner of the square moves here, that angle will grow twice as big. [But] the computer allows everyone to visualize these changes.'
>
> Rubin, A, 1999, p 3

Thus the effect of the dynamic geometry packages may be to enable many more students to engage meaningfully with a range of mathematical concepts that were formerly restricted to the much smaller group who were 'able to supplement…. [static] pictures with some sense of motion', by '[allowing] *everyone* to visualize these changes'. Furthermore, the support that the computer gives to these internal, mental visualisations encourages their development. With time, many more students whose visualisation skills would have atrophied and faded in a traditional, text-book based environment are likely to improve their ability to think visually.

Another type of activity that may be opened up by ICT to a wider audience is mathematical simulation. As Rubin explains,

> ‘There are certain natural systems (most popularly, predator/prey systems) for which the basic structure is expressed by a relatively compact set of rules but the behavior can be vastly different depending on the value of a few variables such as the birth and death rates of the predators. Having a tool with which to explore these patterns not only gives students the opportunity to learn about a biological interaction, but teaches them about functions, variables, cyclical functions and sensitivity analysis. Exploring these concepts by hand is practically impossible, since the number of calculations necessary to see any kind of pattern is astronomical. A whole new area of mathematics is suddenly available to middle and high school students.’
> Rubin, A, 1999, pp 12-13

So here again, an area of mathematics that used to be the province of a few advanced mathematicians has been opened up by the use of ICT to a much greater number of younger and less experienced students.

Pedagogy and content

So ICT clearly has much to offer to teachers who are able to access and use it in the classroom. As Douglas Butler observes,

> ‘There is plenty of anecdotal evidence that teachers can cover topics in less time (and more effectively) when ICT is used,’

although he acknowledges that

> ‘This is not an easy point to prove with formal research’
> (Butler, D, 2005, p 125)

But here the impact of ICT seems to relate, not so much to *what* students are taught, but to *how* they are taught. On the other hand, our main interest, as test developers, is on the content rather than the pedagogy – on the *what*, not the *how*. Our key question is: Do students who study mathematics in an ICT-rich environment actually learn something different to those who follow a totally print-based curriculum? Or do they learn the same things – more quickly and effectively perhaps, but with no essential difference to the concepts being learnt? This is significant, because if the nature of what is learnt is essentially the same in the two cases then there is no reason why we should not go on using the same assessment methods – the same kinds of tests, examinations and teacher assessment activities – that we have always used, however our students have been taught. On the other hand, if the students in the ICT-rich classroom are learning something that is qualitatively different – so that they develop a different set of key mathematical concepts – then these different concepts ought to be assessed.

But here the picture suddenly becomes much more complicated. It is very hard to define what it is that students learn with, or without, computers. Rather, there is a continuum, with a lot of overlap.

So, for example, we have seen that a significant aspect of an ICT-based mathematics curriculum is the ready availability of dynamic images. But dynamic images are nothing new. Some – though certainly not all – teachers have been using them, and writing about them, for years. The approach to transformational geometry in the first School Mathematics Project (SMP) 'O' level course in Britain in the late 1960s, for instance, laid great stress on the development of what was then called students' 'geometrical intuition' – which is very close to what we might now refer to as their ability to visualise. Students were introduced to mathematical translations in Book 2 (for higher-achieving students aged 12 to 13 years), for example, with the explanation,

> 'When an object is translated each point of the object undergoes the same change of position because it moves the same distance in the same direction as every other point. When you see a platoon of soldiers drilling, a formation team dancing, or a pair of ice skaters figure skating and see them moving as a single body it is because each person involved moves in exactly the same way.'
> (SMP 1970, pg 120-121)

This picture of a group or a pair 'moving as a single body' is a strong dynamic mental image that supported students' understanding of the concept of a mathematical translation. In later sections of the book copious exercises with tracing paper, mirrors, scissors and drawing pins were used to give students plenty of experience of the movements involved in such transformations as translations, reflections and rotations. These dynamic images fostered the mental visualisations that students needed if they were to make sense of transformational geometry.

The dynamic images of *Cabri Geometre* or *Geometer's Sketchpad* are a direct descendent of all those practical and kinaesthetic exercises of forty or more years ago. What the computer does is not something completely new. It just makes it much easier to create, manage and share such images. ICT gives the student and the teacher greater access and more control, but some teachers, at least, have always offered powerful images to enable students to develop their understanding of mathematical concepts. ICT can encourage and support the development of teachers' and students' visual imagery, but the imagery itself has been around, in the minds of some mathematicians and some teachers, since Pythagoras, perhaps – long before geometry was pinned down in printed text.

Opening the door to dynamic imagery

What ICT can do, however, is to open the door to dynamic imagery to a much wider audience of teachers and students. By making them so much more accessible, computers can bring the images into many more classrooms. However, as with any door, opening it is one thing: persuading teachers to go through it is another. As Kenneth Ruthven explains, any new technology

> 'is likely to be treated as a variant or hybrid of those that are better established and more familiar. When a new technology is assimilated to established methods …., it functions as an 'amplifier' of existing forms of action.'
> (Kenneth Ruthven, Sara Hennessy, Rosemary Deaney, in press)

In other words, at least to begin with, any new technology is likely to be used to do much the same sort of thing, in much the same way, as the old technology. There are many examples– the QWERTY keyboard that was Papert's despair, for example, or the 'on screen protractors' provided in some primary mathematics teaching software packages (Papert, p 33; Key Press, http://www.keymath.com/x3311.xml). Many teachers may use the new technology to go on teaching the old, print-based curriculum – more efficiently and more effectively, perhaps, but not, in the end, innovatively.

The obstacles that can face attempts to change any aspect of school-based education have been remarked on many times. Noss and Hoyles (1996) summed up their pessimism in relation to mathematics education:

> 'The post-war period has been replete with attempts to reform children's mathematical learning. Interestingly enough, many if not all of the initiatives were catalysed by a similar set of slogans: the curriculum is out of date; it does not reflect new ideas about mathematics and about the way it is learned, it does not help children develop their full mathematical potential; it produces negative attitudes to the subject among certain groups (such as girls and minorities); and finally, because of the importance of mathematics in today's society, the poor competence in mathematics amongst the population at large has far-reaching consequences for national wealth and competitiveness. A 'solution' is then proposed…. Whatever the solution, its standard trajectory begins with panacea and ends with disappointment. '
> (Noss & Hoyles, 1996, pp 156-7)

This is all very depressing. But perhaps the difference in the case of the digital curriculum lies in the cultural change that is taking place in the wider society, quite independently of schools and the formal educational structure. Many of the students in British classrooms now are 'digital natives': they became familiar with computers before they acquired such school-based skills as the ability to read a book or write with a pencil (Prensky, M, 2001). Computer literacy is what they grew up with: traditional print-based literacy came later. An increasing number of newly qualified teachers who are now entering the teaching profession have had similar experiences. The reign of the 'digital immigrants', whose education took place entirely or mainly before computers were commonplace in schools and homes, is inevitably limited. It is perhaps difficult for those of us who were never taught to type at school, who cannot text properly, or who still turn to a reference book rather than a search engine when we want to check a fact or find some information, to realise just how inappropriate and inefficient our habits can seem to a digital native – but change, I would argue, is inevitable, in the classroom as much as in the home or the work place. Eventually, as the digital natives take over positions of responsibility, the school curriculum, including the mathematics curriculum, will develop to support a digital-based, not a text-based, learning environment.

Assessment and change

One key factor that will determine the speed with which ICT will be embraced by the teaching community is the nature of formal assessment. Teachers are, of course, duty bound to ensure that their students receive the best possible preparation for the tests and examinations that will determine their access to further education or their progress in the world of work. If a practice is not allowed in the examination hall then it will not be taught, no matter how relevant it may be outside school. So, for example, David Wright discusses the valuable contribution that graphical calculators can make to the teaching and learning of mathematics. But, as he explains,

> 'In England, assessment policy for their use in A-level examinations has been inconsistent and has led to a reduced level of usage by students and teachers.'
> (Wright, D, 2005, pg 146)

But even when the assessment structure itself is used to drive a change in pedagogy – on the grounds, perhaps, that 'What You Test Is What You Teach' (WYTIWYT) – the demand for a rigid marking scheme that can guarantee consistency across markers may undermine the innovators' best intentions. Noss and Hoyles describe how, following the Cockcroft report, *Mathematics Counts* (Cockcroft, 1982), there was a move in the UK to

> 'introduce investigative learning into mathematics classrooms'.
> (Noss and Hoyle, op cit, p157)

As they explain,

> 'The main vehicle by which this change was to be introduced was through extended coursework in GCSE mathematics for 16-year-old students'.
> (Noss and Hoyle, op cit, p157)

But this attempt to use the assessment structure to create a change in the pedagogy foundered on the rock of marker reliability. As the authors explain,

> '…largely as a consequence of the requirements of assessment, investigations became institutionalised and lost their investigative character. The ubiquitous data-pattern-generalisation investigation which reduced all mathematical situations to numerical models and sought no justification beyond numerical argument, became drills of a new kind – in direct opposition to the original intentions of the reform and in the process lost all their mathematical integrity.'
> (Noss and Hoyle, op cit, p157)

Thus the challenge is not only to identify aspects of the digital mathematics curriculum that cannot be assessed as effectively with a pen-and-paper test as with an ICT-based assessment, but also to devise computer-based activities that exploit the potential of the computer, and are not stripped of all their richness and value 'as a consequence of the requirements of assessment'. Not an easy task for the test developer!

Assessing a digital mathematics curriculum

So – if we want to develop an assessment structure that really does support and encourage the development of a digital mathematics curriculum in our classrooms, how might we go about it? What might we seek to assess, and how?

If, as has been argued here, mathematical models and dynamic images constitute one of the hall marks of the digital mathematics curriculum then the students' ability to understand and to use these images may, perhaps, be regarded as at least one valid focus for the assessment of that curriculum. This is an approach that we have adopted at the NFER in an attempt to develop a series of assessment instruments for students aged 9 to 11 years in the last two years of primary school. These digital tests are designed to support and encourage, rather than undermining or subverting, the effective use of computers in the mathematics classroom. In this series of tests the computer is first used to do what it is so good at – to provide a set of stimulating, engaging and enlightening dynamic models relating to the topic being assessed. Then the program goes on to assess the students' ability, not just to answer questions on the topic, but to learn, and to actually develop their conceptual understanding in the course of the assessment.

At present the model is very crude, and essentially consists of a single structure. A concept is introduced, using dynamic images. Then a question is asked. If the student answers the question correctly they gain two marks. But if they cannot answer it then they are given further teaching in the form of a 'help' screen, and are invited to make a second attempt to answer the question. If they get it right this time then they are awarded one mark. Whether or not they got the question right at the first or second attempt, the concept is then explained again, in the context of the question that has been asked, before the student moves on to the next, related but more challenging, question. (See Fig 2.)

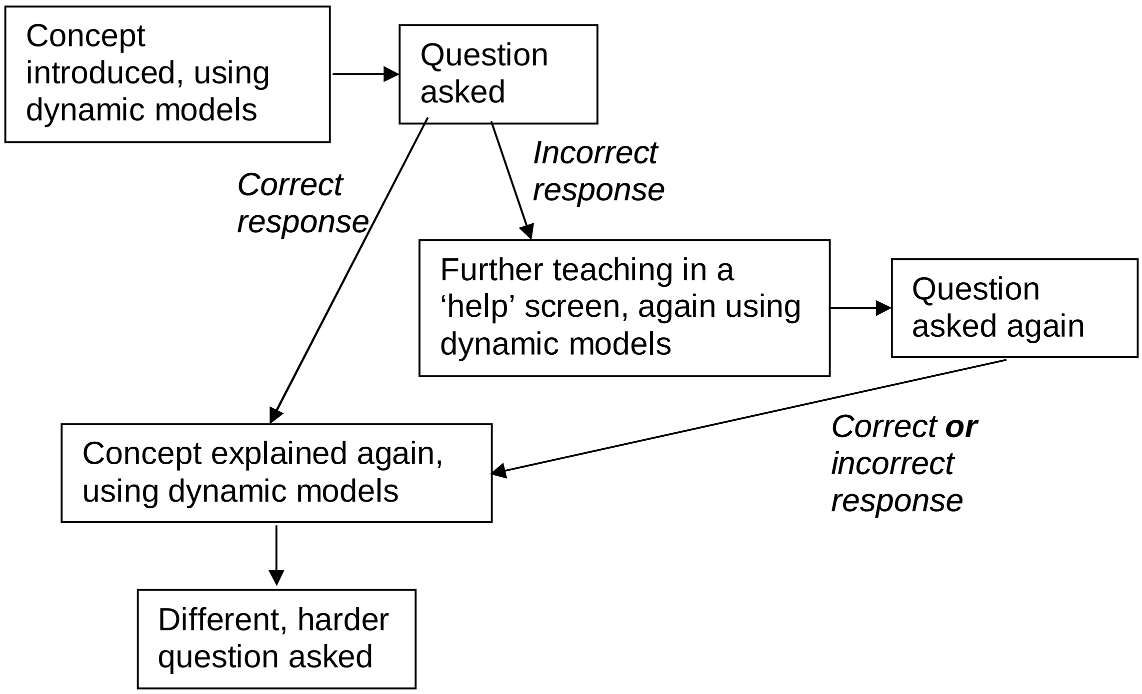
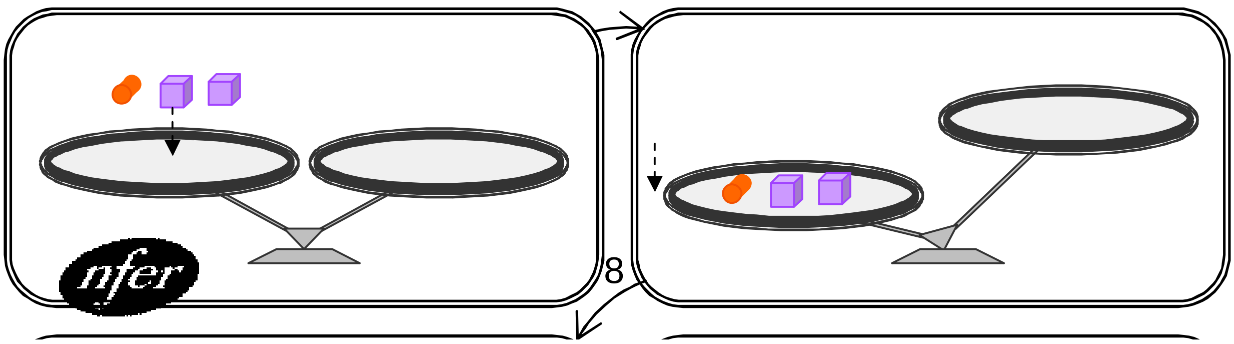


Fig 2 The structure of the digital mathematics test questions

So, for example, an assessment activity focusing on Algebra introduces the concept of an equation with a simple example, *2p + 1 = 5*. The dynamic image used to explain this concept involves a balancing scale. The model of the scale, with *2p + 1* on one side, and *5* on the other, is built up step by step. (See Fig 3.)



Then the student is taken through the process of solving the equation, modelled as the removal of equal quantities from each side of the balancing scale in turn. The student is instructed to *Click the button to subtract 1*, first from the left-hand side, and then from the right hand side. (See Fig 4.)

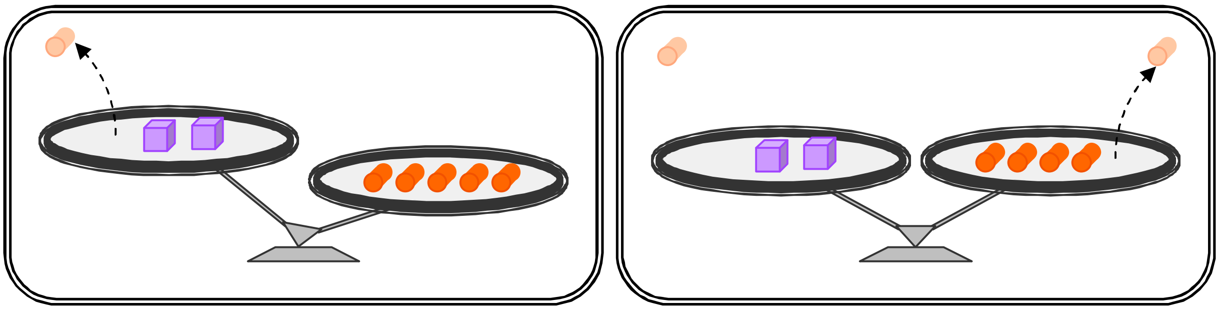


Fig 4 Subtracting 1 from each side

This gives a model of the simplified equation, $2p = 4$. The next step is to divide each side by 2, which is achieved by first splitting the objects remaining on each side of the scales into to two equal sets, and then removing half of the objects from each side. (See Fig 5.)

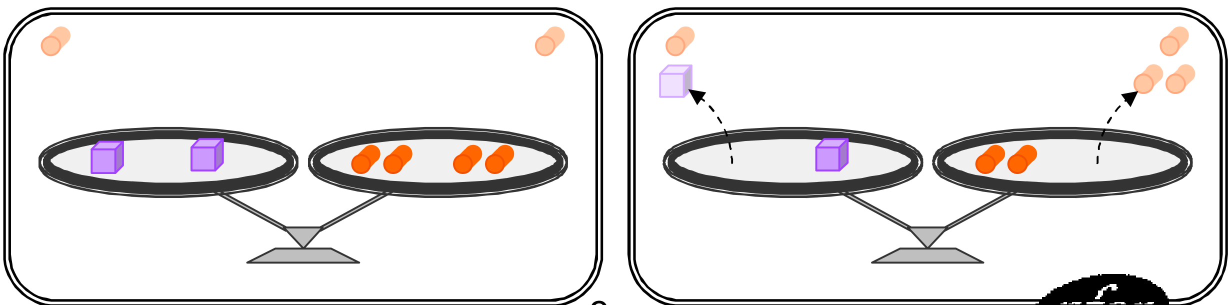
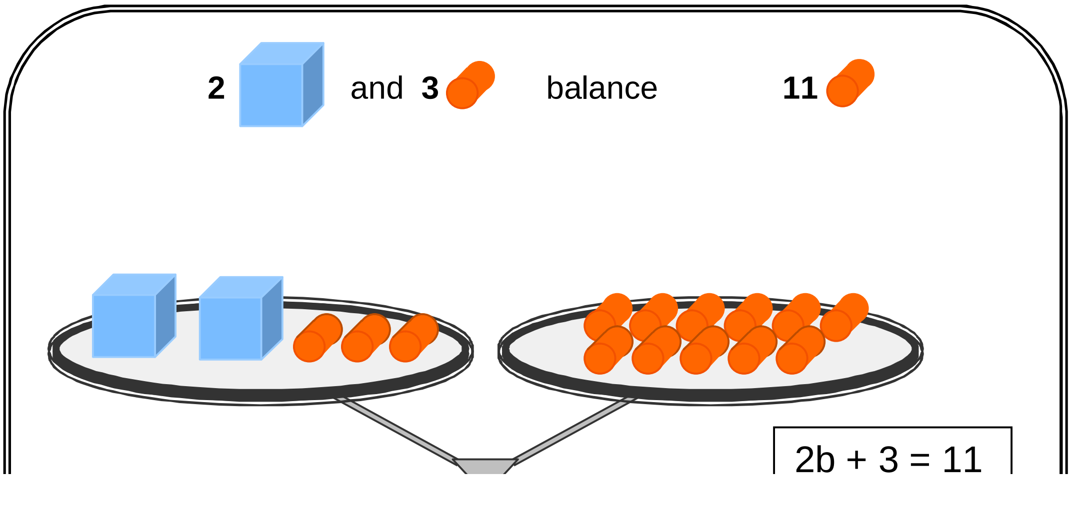


Fig 5 Dividing each side by 2

This gives a graphical image of the solution to the equation, *p = 2*.

A balancing scale is a mental image that has been used in some text books for many years (e.g. see Heylings, MR 1982, pp 8-12). What the computer graphics offer, however, is the dynamic element, which brings the process of balancing the equation to life by allowing the student to watch the elements being rearranged or removed, step by step, on each side of the equation. This can help to establish a clear understanding of the nature of an equation, in which everything on one side must have the same value as everything on the other. This insight is essential if students are to develop a sound grasp of complex algebraic equations, but it can be undermined all too easily by a confusion caused by students' familiarity, early in their school career, with the equals sign '=' (Clausen-May, 2005, p 54). Faced with an exercise of calculations such as 3 + 4 = , or 241.4 ÷ 17 = , or $\sqrt{(20449)} + 31^3$ = , or whatever, students may interpret the equals sign to mean *Work out the calculation on the left, and write down the answer on the right*. When they come to solve algebraic equations such as n + 5 = 17, or 3*n* + 4 = 10, this failure to appreciate the meaning of the equals sign may be compounded by such rote-learnt rules as 'change sides, change signs'. On the other hand, the computer allows us to show the image of the balance dynamically, and thus, perhaps, to support the understanding of an equation as a balance for a greater number of students.

Having introduced the concept of an equation as a balance, with the image of the balancing scale to support it, the program goes on to ask the student to solve a different simple equation, *2b + 3 = 11*. A static image of the balancing scale is shown, but no further help is given at this stage. (See Fig 6.)



Now if the student answers the question correctly their understanding is confirmed with an animation of the process of removing 3 from each side, and then dividing the remaining objects on each side into two parts and removing one of them. If students are unable to solve the equation by themselves, however, they are shown only the first step in the solution. (See Figs 7a and 7b.)

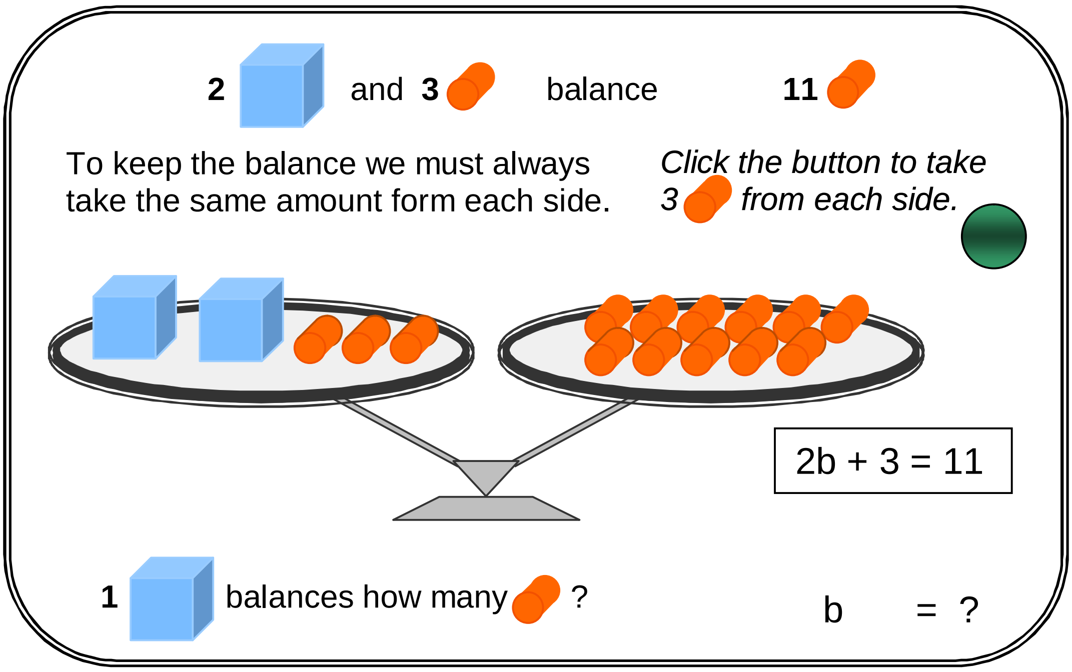
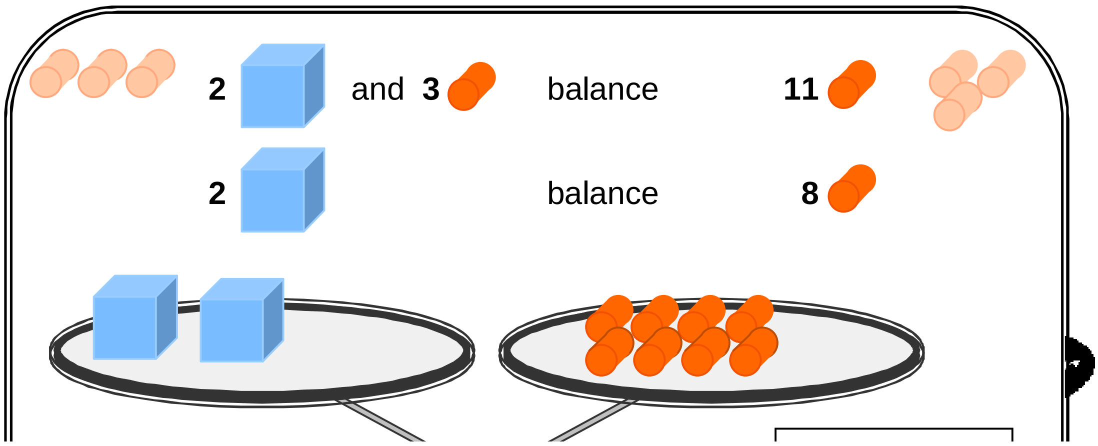


Fig 7a Subtracting 3 from each side



Then they are offered a second opportunity to find the solution to the first equation, before moving on to a second similar but more challenging problem.

This model is being used to develop tests that cover a range of topics, including simple arithmetic, fractions, decimals, angles and rotations, and position and movement. In each test, data may be collected not only on the students' success or failure with each question but also on the development of their understanding during the course of the assessment itself. A student may struggle when the first of a pair of questions is presented for the first time, but then, having worked though the 'help' screen, they may go on to answer it correctly. Then they may find that they can answer another more difficult question on the same topic at the first attempt. Such a student has shown evidence, not just of their mathematical knowledge and understanding, but, perhaps more importantly, of their ability to learn – to understand and apply a new concept. This may be regarded as evidence, not just of rote-learnt knowledge, but of mathematical thinking of a different, and significant, kind. The mathematical models and dynamic images that the computer makes available may, perhaps, constitute at least one aspect of the digital mathematics curriculum – and in order to assess the student's understanding of these, we may need a new kind of test.

The research on which tests like the one described above are based is still in its infancy, and the materials have yet to be fully trialled in the field. None the less, although the phrase 'assessment for learning' has recently been attached to almost any assessment materials that can be used for diagnostic or formative purposes, the hope is that what has been described here may, perhaps, be regarded as a small step towards Black and Wiliams' original objective that

> 'The feedback [on tests] should give each student guidance on how to improve. '
> (Black, P and William, D 1998, p13)

The programs are designed both to teach and to test, so that the interface between assessment and learning is deliberately blurred for the student. The reporting allows the teacher to find out, not just what the student knows, but also, and perhaps as significantly, how effectively they can learn a new concept. The tests offer the possibility, at least, of a style of assessment that does, indeed, go beyond the traditional paper and pencil test to assess something different.

References

Black, P. and William, D. (1998) *Inside the Black Box*. London: King's College.

Butler, D. (2005). 'The school mathematics curriculum in a technological age.' In: Johnston-Wilder, S. and Pimm, D. (Eds) *Teaching Secondary Mathematics with ICT*. Milton Keynes: Open University Press.

Clausen-May, T. (2005). *Teaching Maths to Pupils with Different Learning Styles*. London: Paul Chapman Publishers.

Cockcroft, W.H. (1982). *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools Under the Chairmanship of Dr. W. H. Cockcroft.* London: Department of Education and Science.

Heylings, M.R. (1982). *Algebra*. London: Schofield and Sims.

Johnston-Wilder, S and Pimm, D. (Eds) (2005). *Teaching Secondary Mathematics with ICT.* Milton Keynes: Open University Press.

Key Curriculum Press (2007). 'Protractor.' In: Key Curriculum Press *Dynamic Geometry*. Third edn. [online]. Available: http://www.keymath.com/x3311.xml [26 September, 2007].

Mackrell, K. and Johnston-Wilder, P. (2005). 'Thinking geometrically.' In: Johnston-Wilder, S. and Pimm, D. (Eds): *Teaching Secondary Mathematics with ICT.* Milton Keynes: Open University Press.

Noss, R. and Hoyles, C. (1996). *Windows on Mathematical Meanings: Learning Cultures and Computers.* Kluwer Academic Publishers.

Papert, S. (1984). *Mindstorms: Children, Computers and Powerful Ideas*. New York, NY: Basic Books.

Prensky, M. (2001). 'Digital natives, digital immigrants', *On the Horizon*, **9,** 5 [online]. Available: http://www.marcprensky.com/writing [26 September, 2007].

Rubin, A. (1999). *Technology Meets Math Education: Envisioning a Practical Future*. Washington, DC: US Department of Education, Office of Educational Technology.

Ruthven, R., Hennessy, S. and Deaney, R. (forthcoming). 'Constructions of dynamic geometry: A study of the interpretative flexibility of educational software in classroom practice', *Computers & Education.* http://www.elsevier.com/wps/find/journaldescription.cws_home/347/description#description

School Mathematics Project (1970). *Book 2* (Metric edition). Cambridge University Press.

Wright, D. (2005). 'Graphical calculators: tools for mathematical thinking.' In: Johnston-Wilder, S. and Pimm, D. (Eds) *Teaching Secondary Mathematics with ICT*. Milton Keynes: Open University Press.

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